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CAN RICK AVERSION IN FIRMS REDUCE UNEMPLOYMENT PERSISTENCE?

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Abstract

This paper contributes to the growing literature that attempts to explain unemployment persistence. We show that when the economy is struck by a negative transitory (or permanent) demand or supply shock, firms can find their way back quicker to the pre-shock (or new) employment levels if they are risk-averse. The reason is that risk aversion in firms creates a self-adjusting mechanism whereby cautious firms adjust hiring and wage-setting decisions to try to regain the pre-shock employment levels and minimize fluctuations in profits. Therefore, perhaps surprisingly, risk aversion in firms is seen as a stabilizing macroeconomic force that reduces unemployment inertia.

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1 Introduction

The question of why the unemployment rate takes a long time before it reverts back to its natural rate following a negative exogenous supply or demand shock has been the subject of unremitting interest in macroeconomics. An insight into this question is crucial for understanding the rise in the unemployment rates in many of the OECD countries in the last two decades and is equally crucial for the formulation of public policy.

This paper contributes to the debate on the causes of unemployment persistence by raising the interesting theoretical possibility that a lower degree of risk aversion on the part of the firms can obstruct the speed at which employment reverts to its steady-state levels which we also call its natural rate. From the perspective of the intertemporal-equilibrium model of endogenous natural-rate theory of Hoon and Phelps (1992) we show that in the aftermath of a random transitory (or permanent) shock to either productivity or interest rates which drive the economy's employment level below its steady-state, risk aversion in firms tends to accelerate the speed of adjustment back to the steady-state employment. The reason is that risk aversion in firms serves as a self-adjusting device whereby cautious firms try to regain the pre-shock employment levels in an attempt to stay close to the steady-state profit levels. Thus, firms in a such a world like to keep closer to their steady-state profit levels and strive to regain these levels once perturbed from them. However, this close tracking of the 'steady-state' comes at a cost in the form of altering wages, hiring or losing workers during the period in which adjustment takes place. In this sense, risk aversion can be seen as to actually reduce sluggishness in the employment series irrespective of the levels of labour market rigidities characterized in the form of high training cost and low quit rates in our model.¹

Our theory offers a new prospective on the speed of unemployment adjustment. A model that comes close to the ideas developed in this paper is Frank (1990) in that in both papers firms are risk averse; but in contrast to Frank our model is dynamic

¹Low quit rates and high training costs could both result from legislation in the labour markets.

and focuses on adjustment speeds of unemployment (rather than the sustainability of equilibrium recessions).

A crucial empirical question is to what extent can we treat firms as being risk averse? And if so, what makes firms behave in this way? Some factors that can explain the latter question are non-diversified owners, cash constraints and financial problems. Risk aversion in firms is also commonly found in labour market contracting models such as Grossman and Hart (1981). Perhaps a better answer to the latter question and also partly to the first, lies in the literature on agency theory. In particular, theories on incentive of procurement and regulation suggest that the degree of risk aversion amongst decision-makers in firms may actually be generated by the incentive structure in that organization (Laffont and Tirole, 1993, 1986, 2000). For example, take a large modern firm in which there is a separation between ownership and management and the managers maximize their lifetime incomes by pleasing the owners who favour steady income growth. In such a set up managers are likely to avoid risky situations and prefer steady income growth (Monsen and Downs, 1965). One way to ensure that managers act like owners is to offer bonuses in the form of shares or regulate such that managers have to accumulate and then keep a certain number of shares by a certain period. This is often the current practice.² Hence, the degree of risk aversion is endogenously determined by the structure and the incentives provided by the owners of the organization.

The reason why we believe this is interesting in the context of the unemployment debate is that the choice of one institutional structure and incentive schemes over another may emerge from the regulatory environment, political structures and individual and social values within which the firm operates. We would expect such features and hence the degree of risk aversion to differ across countries (and also across time). However, this is not the focus of this paper which is exclusively concerned with the relationship between the speed of adjustment of unemployment and risk aversion.

²See, for example, remuneration reports 2002 for Hilton Group plc, ICI plc, HBos plc and Invensys plc.

The answer to the first question on the empirical relevance of risk aversion in firms can be best found in surveys on the risk-taking behavior of entrepreneurs where a large proportion of established-entrepreneurs who own their businesses desire a moderate level of risk (Mancuso (1975) and Brockhaus (1980)). Interestingly, MacCrimmon and Wehrung (1984, 1990) study a randomly selected sample of top-level executives were asked to play a hypothetical role within a business context and make decisions under a variety of risky situations. It was found that executives had the tendency of avoiding risk or delaying decisions until more information was made available to them. Thus, it seems that risk aversion in firms may be more commonplace than one might normally expect.

The remainder of this paper is organized as follows. In Section 2 we briefly present the related literature. In Section 3 we develop the basic structure of the model and solve for the steady-state. In Section 4 we analyze, using both theory and numerical simulations, the dynamic properties of the model with respect to the firm risk aversion and the speed of adjustment of unemployment. Finally, in Section 4 we present the concluding remarks.

2 Related Literature

The existing literature offers two strands of thinking which try to explain the behaviour of unemployment. One the one hand, 'the persistence school' take the view that changes in unemployment are determined by past shocks, rather than changes in the fundamentals determining the economy which in turn shift the natural rate.³ The emphasis here is on the dynamics of unemployment during the period in which the economy finds itself away from the natural unemployment rates. Particular attention is paid to how the interplay between current and past nominal and real shocks impact on the adjust of the unemployment towards its natural-rate. For example, Henry, Karanassou and Snower (2000) and Karanassou and Snower

³See Turner et al., (2001) for a survey.

(1998 a, b) show that a chain of past transitory but long-lasting shocks to labour demand and supply and real wages delays the adjustment of unemployment to its steady-state levels. The economy, therefore, takes a while before it works its way through these networks of shocks.

According to an alternative strand of thinking Hoon and Phelps (1992), Phelps (1994), Phelps and Zoega (1998) argue that the natural unemployment-rate has a time path that is generated by the structure of the economy: its technological factors, institutions, laws and individual and social preferences. This is commonly referred to as the 'structuralist school'. Here, real disturbances are met by changes to the structure of the economy (labour tax laws, working hours, etc.) which then permanently shift the natural unemployment rate. Thus, the movements from one equilibrium natural-rate to another are considered as the prime source of increased unemployment that we observe in OECD countries. Also the economy can be slow to adjust to these new equilibrium natural rates. There are many reason why the adjustment to the new equilibrium natural-rates can be slow. For example, in Bentolila and Bertola (1990) and Bertola (1990) hold responsible hiring and firing cost for creating inertia in unemployment rate. Indeed, with hiring and firing costs as sunk costs the expected profitability per hiree has to be high enough to justify hiring. However, until this perceived profitability reaches the threshold that justifies hiring, unemployment rates may stagnate. The insider-outsider theory of Lindbeck and Snower (1988) and Blanchard and Summers (1986) argue that in the aftermath of a negative shock to unemployment, the remaining employees - the insiders bargain for higher wages with little regard to the unemployed - the outsiders. The insiders have such bargaining power due to the presence of hiring and firing costs. The consequence of this process is that they are able to push their wages high at the cost of creating employment and again delaying the recovery process. Another plausible reason for inertia is the shortages in capital stock that may prevent firms from hiring rapidly. Thus, the longer the firm takes to install new capital with which new workers can work, the longer unemployment rates persist. Finally, Blanchard and Diamond (1994) and Layard and Nickell (1987) argue that during prolonged durations of unemployment, unemployed workers gradually loose human capital. Not only this makes them less attractive workers to hire but it also erodes their sharpness during the job-search process. Such workers, therefore, take a long while before they get back employment making the overall unemployment levels slow to adjust. Amongst all these explanations, Bean (1994) argues that there is reasonable evidence to suggest that the hiring and firing costs and long durations of unemployment arguments are more successful in explaining the slow adjustment problem.

Against this background, this paper follows the structuralist school, and Hoon and Phelps (1992, 1997) and Orzag and Zoega (1995, 1995) in particular, generalizing the influential core model in this literature to the case where firms are risk averse. Our results show that risk-aversion diminishes the role factors such as hiring and firing costs play in determining the delays in the speed of adjustment to the natural unemployment-rate which occur following a demand or supply shock.

3 The Basic Model

Production is carried out by n identical competitive firms each with labour, E/n, as the only factor of production and with constant returns to scale. n is exogenous. Therefore, the production function is given by

$$Y = f(\frac{E}{n}) = \Lambda \frac{E}{n} \tag{1}$$

where Y is the firm's output, E is total employment and E/n is the stock of employees each firm has. The dynamics of total employment are

$$\dot{E} = H - q(\frac{\bar{w}}{w})E; \ q' > 0, \ q'' \ge 0$$
 (2)

In (2), the change in employment is the difference between the total number of trainees, H, and number of people who quit, qE. The variables Y, H and E are measured in per capita terms such that we normalize the workforce $L \geq E$ at unity.

This implies that E is the employment rate, while H is the number of trainees per capita. As in Salop (1979) and Hoon and Phelps (1992) the quit rate, q, is a concave function of the ratio of the expected average industry wage, \bar{w} to the firm's own wage, w. When the wage prospects are better elsewhere the employees leave the firm at the rate of $\frac{1}{w}q'(\cdot)$, while when the employing-firm offers wages that are better than the industry average the employee quit rate drops by $-\frac{\bar{w}}{w^2}q'(\cdot)$.

At time τ , the representative firm employs the proportion E/n denoted by e, out of the total workforce⁴ and maximizes the present discounted value of a concave function of profits given by

$$V(\tau) = \int_{\tau}^{\infty} u(\pi(t)) \exp(-r(t-\tau))dt$$
 (3)

where $\pi = (\Lambda - w - T(h))e$, T(0) = 0, T', T'' > 0.

Firms are risk averse⁵ and face a concave utility function in profits such that $u'(\cdot) > 0$ and $u''(\cdot) < 0$. In the profit function, π , the first term is the output produced by the firm where prices have been suppressed to unity. The second term is the wage bill of the firm. The final term T(h)e is the total cost of training H new workers, where h = H/E is the firms' hiring rate. The convex function T(h) denotes the training cost per unit of labour and captures the output lost per worker at the hire rate h. The discount factor $\exp(-r(t-\tau))$ is the rate at which output can be substituted intertemporally. The rate of interest r is exogenously given.

⁴The proportion of workers employed by each firm is $\frac{E/L}{n}$. However, given our normalization L=1 we arrive at $\frac{E}{n}=e$.

⁵Strictly speaking since our analysis is deterministic throughout there is no risk in our model and 'risk aversion' refers to a firms with a finite elasticity of intertemporal substitution. We examine the speed of convergence of such firms and the macro-economy to a given steady state which can change as a result of permanent shocks to underlying parameters. In a stochastic environment, of course, we can refer to our firms as indeed being risk averse.

3.1 The Firm's Optimization Problem

At time, τ , the firm maximizes $V(\tau)$ given by (3) with respect to $\{w(t), h(t)\}, t \geq \tau$ subject to e(t) given by

$$\dot{e} = \left[h - q(\frac{\bar{w}}{w})\right]e\tag{4}$$

given e(0) and a terminal condition $\lim_{t\to\infty} [e^{-rt}e(t)] \ge 0$. Equation (4) shows how employment evolves at firm level and is obtained by dividing both sides of equation (2) by the number of firms n. Assume that n is constant.

To solve the firm's maximization problem, we define the current-value Hamiltonian given by

$$\mathcal{H} = \exp(-rt) \left[u(f(e) - we - T(h)e) + \lambda (h - q\left(\frac{\bar{w}}{w}\right))e \right]$$
 (5)

where λ is the shadow value of the trained worker. Then applying Pontryagin's Maximum Principle, the first-order necessary conditions are given by:

$$\frac{\partial \mathcal{H}}{\partial h} = 0 \Rightarrow T'(h)u'(\pi) = \lambda \tag{6}$$

$$\frac{\partial \mathcal{H}}{\partial w} = 0 \Rightarrow u'(\pi) = \frac{\lambda \bar{w}}{w^2} q'(\frac{\bar{w}}{w}) \tag{7}$$

$$\dot{\lambda} - r\lambda = -\frac{\partial \mathcal{H}}{\partial e} = -u'(\pi) \left[f'(e) - w - T(h) \right] - \lambda \left[h - q(\frac{\bar{w}}{w}) \right]$$
 (8)

$$\frac{\partial \mathcal{H}}{\partial \lambda} = 0 \Rightarrow \dot{e} = \left[h - q \left(\frac{\bar{w}}{w} \right) \right] e \tag{9}$$

$$\lim_{t \to \infty} [\exp(-rt)\lambda(t)e(t)] = 0 \tag{10}$$

In Equation (6) the right-hand-side is the value of a trained worker, while the left-hand-side is the sacrifice the firm makes – in the form of the output loss that occurs when trained workers spend time training the hirees instead of actively producing and the marginal change in firm's profits evaluation that results from this investment – in having to train a newly hired worker. Thus, the equation shows that the firm will sacrifice resources on training until it brings the value of a marginal trainee at parity with the value of the trained employee. Interestingly, the intuition just spelled out is subtly different in comparison with Hoon and Phelps (1992) and other studies such as Orszag and Zoega (1996, 1995). Indeed, our model features the

possibility that any variable that affects firm's current profitability also affects the valuation (the utility) the firm assigns to a profit level. This in turn affects the firm's perception of the value of its stock of employees. Consequently, we can infer from our first-order-condition equation (6) that an extra worker is worth substantially more at lower levels of profits compared with higher profit levels due to diminishing marginal utility in profits. This is in contrast to other studies that adopt the usual assumption of risk-neutral firms with utility functions linear in profits, since then the worth of an extra unit of profit is independent of the levels of profits.

Equation (7) tells that at the optimum the marginal cost of raising wages by a unit, in the form of the lower current profits due to the direct wage increases by the amount e and the fall in the marginal utility associated with these reduced profitability level, equals the marginal benefit in the form of lower turnover costs due the retaining of employees whose value is given by λ .

Using (6), equation (8) can be rewritten as

$$\dot{\lambda} = -u'(\pi) \left[f'(e) - T(h) + hT'(h) \right] + \left[\lambda q(\frac{\overline{w}}{w}) + wu'(\pi) + r\lambda \right]$$

and shows that the shadow value of a worker evolves over time, λ , according to the marginal benefits and marginal costs that result from raising employment by one unit. The marginal benefits – the first term – come in the form of the net marginal product of a newly employed worker and the immediate change in the marginal utility in profits that is associated with this extra product. The net marginal product is composed of the marginal product from extra employment, f'(e), the loss in the output caused by our employee having to train trainees instead of actively producing, T(h), and the savings in the marginal costs of training made possible by our new employee sharing the burden of training the trainees, hT'(h). The marginal costs, the second set of terms of the right-hand side, are composed of the direct wage cost of employing a worker and the marginal loss in the utility of profits that this extra cost brings, $u'(\pi)w$, the quit rate evaluated at the worker's shadow value, $\lambda q(\frac{\bar{w}}{w})$, and the opportunity cost of investing on the employee, $r\lambda$, evaluated at the shadow-value of the worker.

3.2 The Macroeconomic Equilibrium

To complete our macro-model of employment, hiring rate and wages we recall that our normalization of the workforce at unity implies that E and 1-E are the employment and unemployment rates respectively. Then having set out the individual firm's optimization problem we can endogenize the expected average industry wage as $\bar{w} = \bar{w} \times$ probability of employment = wE, following Calvo (1979), Salop(1979) and Hoon and Phelps (1992). This equilibrium relationship implies that the wage of the representative firm, w, is larger than the expected industry wage if there is unemployment (i.e., if E < 1).

The analysis of the dynamic system (6) to (9) will be carried out in terms of state variables $e = \frac{E}{n}$ and λ . Upon substituting $\bar{w} = Ew$ and dividing (6) by (7) we obtain wages as function of employment and the hiring rate

$$w = Eq'(E)T'(h) = \omega(E, h) \tag{11}$$

say. Substituting into (6) we then have

$$u'(f(\frac{E}{n}) - \omega(E, h)\frac{E}{n} - T(h)\frac{E}{n})T'(h) = \lambda$$
(12)

Hence solving (12) implicitly for h and substituting back into (11) we arrive at w and h expressed in terms of E and λ :

$$h = h(E, \lambda) \tag{13}$$

$$w = \omega(E, h(E, \lambda)) \equiv w(E, \lambda)$$
 (14)

say. Using (14), (13) and (6), the dynamic system (8) and (9) in (E, λ) space can now be written as

$$\dot{E} = [h(E,\lambda) - q(E)] E \equiv F(E,\lambda) \tag{15}$$

$$\dot{\lambda} = \left[r - h(E,\lambda) + q(E) + \frac{1}{T'(h(E,\lambda))} (\Lambda \frac{E}{n} - w(E,\lambda) - T(h(E,\lambda)) \right] \lambda$$

$$\equiv G(E,\lambda) \tag{16}$$

say, plus the transversality condition, (10). One state variable, E, is predetermined at time t with initial condition E(0) given exogenously. The second state variable,

 λ , is non-predetermined or 'free' and satisfies the terminal condition (10) with $e(t) = \frac{E(t)}{\pi}$.

We follow Hoon and Phelps (1992), section III, in interpreting this macro-model with a exogenous constant interest rate as that of a small open economy facing an exogenous world interest rate. Current account dynamics are excluded by conducting our stability analysis in the vicinity of a steady state in which trade is balanced and net foreign assets are zero.⁶

3.3 The Steady State

In the steady state with $\dot{E} = \dot{\lambda} = 0$ we have

$$h = q(E) (17)$$

$$w = Eq'(E)T'(h) = Eq'(E)T'(q(E))$$
 (18)

$$r = \frac{1}{T'(h)} \left(\Lambda - w - T(h) \right) \tag{19}$$

giving us three equations for h, w and E. Then λ is given by (6).

We first examine the wage-employment relationship given by (18). At the steadystate the wages depend on the investment made on the marginal worker— in the form of training costs — and their marginal quit propensity. Differentiating (18) with h = q(E), we then have

$$w'(E) = (q'(E) + Eq''(E))T'(q(E)) + Eq'(E)T''(E) > 0$$
(20)

since T', T'', q'(E), $q''(E) \ge 0$. The first term is the rise in turnover as employment rises, q' + Eq'', and each quitting worker must be replaced at cost, T'. The second term is the rise in training costs associated with higher employment. Thus, higher wages and higher (lower) employment (unemployment) go hand in hand and this relationship is concave (convex) from the origin. This ensures that our model is

⁶Without these simplifying assumptions involving a constant real interest rate and a linearization in the vicinity of a balanced trade steady state, we would be faced with a third-order, analytically intractable dynamic system.

consistent with the Phillip's curve wage-employment (or wage-unemployment) relationship. To summarise:

Proposition 1.

In the steady state, the wage is an increasing function of the employment rate.

The steady-state is at the intersection of the $\dot{E}=0$ curve, (17), and the $\dot{\lambda}=0$ curve. The former is upward-sloping in (h, E) space since q'>0. To show that the latter is downward-sloping, differentiate (19) implicitly to obtain along this curve

$$\frac{dh}{dE} = -\frac{w'(E)}{T' + rT''} < 0 \tag{21}$$

assuming the conditions for proposition 1 hold (in which case w'(E) > 0), and recalling that T', T'' > 0. The derivative implies that, higher levels of steady-state employment are associated with lower levels of hiring rate. Finally we observe that the steady state given by (17) to (19) is independent of the functional form $u(\cdot)$. We summarize these results as:

Proposition 2.

There exists a unique steady state that is independent of the degree of risk aversion of the firm.

Figure 1 shows the 'E stationary' curve ($\dot{E}=0$) and the ' λ - stationary' curve ($\dot{\lambda}=0$) in (h, E) space for functional forms and parameter values discussed in section 3.1. Notice that the equilibrium unemployment rate stands as 5%. In the dynamic analysis that follows, we consider permanent shocks to underlying parameters that change the steady state. We then examine the subsequent dynamic transition from the old to the new steady state. These are discussed in section 3.2.

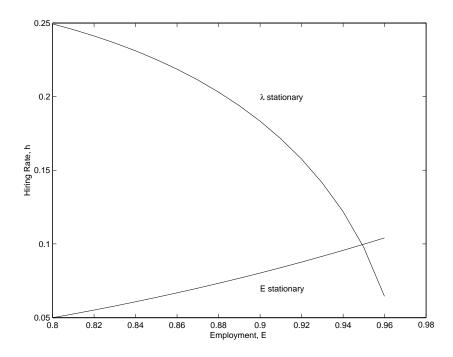


Figure 1: The Steady State in h-E Space

4 Dynamic Analysis

4.1 Saddlepath Stability

To analyze the local stability of the dynamic system (15) and (16) in the vicinity of the unique steady state, we evaluate the Jacobian

$$J = \begin{pmatrix} F_E & F_\lambda \\ G_E & G_\lambda \end{pmatrix} \tag{22}$$

where partial derivatives $F_E \equiv \frac{\partial F}{\partial E}$, etc are evaluated at the steady state. The eigenvalues $\mu = \mu_1$ and $\mu = \mu_2$ are given by the solutions to

$$\mu^2 - (F_E + G_\lambda)\mu + \Theta = 0 \tag{23}$$

where $\Theta \equiv F_E G_{\lambda} - F_{\lambda} G_E$. Since we have one predetermined and one non-predetermined variable, saddlepath stability requires that one root has real part positive and the other has real part negative.

Partially differentiating $F(E,\lambda)$ and $G(E,\lambda)$ defined in (15) and (16) at the steady state and using (17) and (19), we have

$$F_E = E(h_E - q') (24)$$

$$F_{\lambda} = Eh_{\lambda} \tag{25}$$

$$G_E = \lambda \left[h_E - q' + \frac{1}{T'} (w_E + h_E (rT'' + T')) \right]$$
 (26)

$$G_{\lambda} = \lambda \left[h_{\lambda} + \frac{1}{T'} \left(h_{\lambda} T'' r + w_{\lambda} + T' h_{\lambda} \right) \right]$$
 (27)

where q = q(E), T = T(h), f = f(E) and we omit the arguments in q', T', T'' and f''. It follows that

$$\Theta \equiv F_E G_\lambda - F_\lambda G_E = \frac{E\lambda}{T'} \left[-q' h_\lambda (T' + T''r) + w_\lambda (h_E - q') - h_\lambda w_E \right]$$
 (28)

This expression can be simplified further by partially differentiating (11) to obtain

$$w_{\lambda} = Eq'T''h_{\lambda} \tag{29}$$

$$w_E = (Eq'' + q')T' + Eq'T''h_E (30)$$

These partial derivatives imply that, other things being equal, the higher the shadow-value of a worker or the employment rate, the higher wages that has to be paid to the employees at the equilibrium. Such a policy will help deter workers from quitting and hence reduce firms' turnover and training costs.

Hence (24) becomes

$$G_{\lambda} = \lambda h_{\lambda} \left[2 + \frac{T''}{T'} \left(r + Eq' \right) \right] \tag{31}$$

Substituting back into (28) we arrive at

$$\Theta = \frac{E\lambda h_{\lambda}}{T'} \left[-q'(2T' + rT'' + Eq'T'') - Eq''T' \right]$$
(32)

To complete the dynamic analysis we require partial derivatives⁷

$$h_{\lambda} = \frac{1}{u'(\pi)T'' - \frac{u''(\pi)}{n}ET'(Eq'T'' + T')}$$
(33)

$$h_E = -\frac{u''(\pi)(T')^2(r - E(Eq'' + q'))}{u'(\pi)T'' - \frac{u''(\pi)}{r}ET'(Eq'T'' + T')}$$
(34)

⁷These derivations use (11) and (12).

We can now use these results to examine the saddlepath stability of the dynamic system. From (23) the eigenvalues are given by

$$\mu = \frac{F_E + G_\lambda \pm \sqrt{(F_E + G_\lambda)^2 - 4\Theta}}{2} \tag{35}$$

Hence one root $(\mu = \mu_1)$ is real and negative and one root $(\mu = \mu_2)$ is real and positive if $\Theta < 0$. From (34) $h_{\lambda} > 0$, and hence from (32), since $\lambda > 0$, we have that $\Theta < 0$ iff [-q'(2T' + rT'' + Eq'T'') - Eq''T'] < 0. Since we have made the usual convexity assumption that q', q'', T', T'' > 0, all the terms in this expression are negative and hence we have shown that

Proposition 3.

The dynamic system is saddlepath stable.

4.2 The Impact of Macroeconomic Shocks

In the section we study the impact of aggregate demand and supply shocks. First, we analyse a shock to aggregate demand arising from an exogenous fall in the world real interest rate engineered by central banks outside the control of our economy. Second, we study shocks to aggregate supply arising from exogenous productivity shocks. As in figure 1 we remain in (h, E) space (although the dynamic analysis is in (λ, E) space).

Recall that the interest rate appears in (19) and it represents the opportunity cost of investing on an employee. Assuming that we are initially at the steady-state (point A in Figure 2), a fall in the real interest rate implies that $\dot{\lambda}$ curve shifts to the right from (16) and also tilts upwards from (21). The firms now find themselves in a position where the stream of marginal cash flow attributable to an extra employee is too high relative to her marginal opportunity costs of employment. This implies that shadow value of a worker must have jumped with a corresponding rise in investment on new workers. The firm therefore increases hiring causing the training cost and wage cost (from (20)) to climb until marginal profit contribution of an extra employee comes back at par with the marginal opportunity cost of

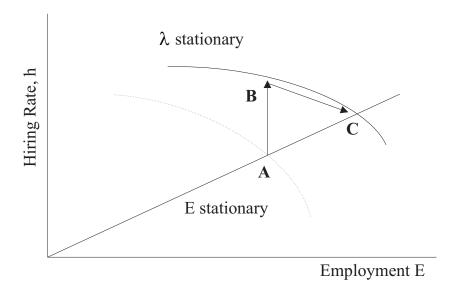


Figure 2: Macroeconomic Shocks

employing her. This story can be seen from the arrows of motion in figure 2. At first the hiring rate jumps to point B since the shadow value of the workers jumps. As more people are employed the rise in training costs and wage bills increase, this reduces the shadow-value of a worker eventually bringing the growth in employment to a halt (travelling from B to C). The natural employment rate permanently rises to point C.

The second shock we study is a permanent positive shock to productivity. We analyze this shock and follow Hoon and Phelps (1992) by interpreting the training costs as some fraction, τ , of a trained worker's productivity Λ . Thus, $T(h) = \tau(h)\Lambda$, where the fraction of productivity that is consumed by training new workers is determined by the level of hiring. In the dynamic system the curve $\dot{E} = 0$ is not affected and we can rewrite (21) as:

$$\frac{dh}{dE} = -\frac{w'(E)}{\Lambda(\tau' + r\tau'')} < 0$$

Now from the equations above a rise in productivity has a very similar affect as the permanent fall fall in the real interest in that the $\dot{\lambda}=0$ curve shifts and tilts to the right. However, this time the marginal profit contribution of an employee instantaneously increases and as a result λ jumps up with a corresponding rise in

investment in new workers and the hiring rate h. As the hiring process continues, training costs and wage bill rise and this serves to pull back the marginal cash flow contribution of the marginal worker to the same level as the opportunity cost of employing her. Thus, the value of h gradually declines and the steady-state employment permanently achieves higher levels.

It is clear from the both shocks that risk-aversion does not play any role in determining the level of the steady state. However, in the next section we show that when the economy is in the state of transition from one steady-state to another, i.e., follow the arrows of motion in figure 2, risk-aversion plays a leading role for either type of shocks.

4.3 Hysteresis and Risk Aversion

Up to this point we have shown that the firm's risk aversion neither affects the steady state nor the possibility of instability (or indeterminacy⁸). However the degree of risk aversion does affect the *size* of the eigenvalues and hence the rate at which the system reverts to the steady state following an exogenous shock. From standard analysis of linear systems with non-predetermined variables (see, for example, Currie and Levine (1993)) the solution to the linearized dynamic system

$$\begin{bmatrix} \dot{E} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} F_E & F_\lambda \\ G_E & G_\lambda \end{bmatrix} \begin{bmatrix} E \\ \lambda \end{bmatrix} \tag{36}$$

is given by

$$\lambda - \bar{\lambda} = -M_{22}^{-1} M_{21} (E - \bar{E}) \tag{37}$$

$$\dot{E} = [F_E - F_\lambda M_{22}^{-1} M_{21}] (E - \bar{E}) \tag{38}$$

where $\bar{\lambda}$ and \bar{E} denote steady-state values and M is a matrix of eigenvectors defined by

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} F_E & F_{\lambda} \\ G_E & G_{\lambda} \end{bmatrix} = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$
(39)

⁸Indeterminacy and multiple equilibria occurs in our model if both eigenvalues have negative real parts. Both instability and indeterminacy are ruled out by proposition 3.

where μ_1 and μ_2 are the real negative and positive eigenvalues found in the previous subsection. Solving (39) we find that

$$M_{22}^{-1}M_{21} = \frac{\mu_2 - G_\lambda}{F_\lambda} = \frac{F_E - \mu_1}{F_\lambda} \tag{40}$$

Hence (38) becomes

$$\dot{E} = [F_E + G_\lambda - \mu_2]E = \mu_1(E - \bar{E}) \tag{41}$$

using trace(J) $\equiv F_E + G_\lambda = \mu_1 + \mu_2$.

We have now established that the rate at which employment returns to the steady steady following either a temporary shock to employment itself (in which case the economy reverts to the same steady state) or to a permanent change in the natural rate of employment (then reversion is to a new steady state) is given by the absolute size of the negative eigenvalue μ_1 :

$$\mu_1 = \frac{F_E + G_\lambda - \sqrt{(F_E + G_\lambda)^2 - 4\Theta}}{2} \tag{42}$$

Provided that the dynamic system is saddlepath stable, which we have shown to be the case, the dynamics of employment is given by

$$E(t) = E(0)e^{\mu_1 t} (43)$$

To see how μ_1 changes as risk aversion increases, consider a constant coefficient of relative risk aversion profit function

$$u(\pi) = \frac{\pi^{1-\rho}}{1-\rho}; \ \rho > 0, \ \rho \neq 1$$

= $\log(\pi); \ \rho = 1$ (44)

where $\rho = -\frac{\pi u''}{u'}$ is the constant coefficient of risk aversion.⁹ In the steady state, using (19) we have that economy-wide profits are $n\pi = rT'E$. Then in terms of ρ ,

⁹Again, as noted previously, in our deterministic environment ρ is strictly the inverse of the intertemporal rate of substitution. In a stochastic environment we can refer to risk aversion.

from (32), (33) and (31) we have

$$F_E(\rho) = \left(\frac{\rho T'(r - E(Eq'' + q'))}{\rho (Eq'T'' + T') + rT''} - Eq'\right)$$
(45)

$$G_{\lambda}(\rho) = \frac{rT' \left[2 + \frac{T''}{T'} (r + Eq')\right]}{\rho(Eq'T'' + T') + rT''}$$
 (46)

$$\Theta(\rho) = \frac{rE\left[-q'(2T' + rT'' + Eq'T'') - Eq''T'\right]}{\rho(Eq'T'' + T') + rT''}$$
(47)

We are interested in the sign of $\frac{d\mu_1}{d\rho}$. If it is positive, them increasing risk aversion pushes μ_1 towards zero and towards pure hysteresis. If the derivative is negative, it has the opposite effect and risk aversion actually improves the stability of the dynamic system. Differentiating (42) we have

$$\frac{d\mu_1}{d\rho} = \frac{1}{2} \left(\frac{dF_E}{d\rho} + \frac{dG_\lambda}{d\rho} \right) \left(1 - \frac{(F_E + G_\lambda)}{\sqrt{(F_E + G_\lambda)^2 - 4\Theta}} \right) + \frac{2}{\sqrt{(F_E + G_\lambda)^2 - 4\Theta}} \frac{d\Theta}{d\rho}$$
(48)

Differentiating (32) we have that

$$\frac{d\Theta}{d\rho} = (Eq'T'' + T')\frac{rE\left[q'(2T' + rT'' + Eq'T'') + Eq''T'\right]}{(\rho(Eq'T'' + T') + rT'')^2} > 0$$
(49)

and so the last term in (48) is positive. Since $F_E + G_{\lambda} < \sqrt{(F_E + G_{\lambda})^2 - 4\Theta}$ the first term has the sign of $\frac{dF_E}{d\rho} + \frac{dG_{\lambda}}{d\rho}$. Consider first derivatives in the vicinity of $\rho = 0$. Differentiating (45) and (46), at $\rho = 0$ we find that

$$\frac{d(F_E + G_\lambda)}{d\rho} = -Eq' - \frac{1}{r(T'')^2} \left[(Eq'T'' + T')(2T' + T''Eq') + T''T'E(Eq'' + q') \right]$$
(50)

which is negative. It follows that in the vicinity of $\rho = 0$, the effect of increasing risk aversion on the degree of stability of the macro-economy is ambiguous. However we can establish an unambiguous result for higher values of ρ , the case where $r << \rho \bar{q}$, \bar{q} being the quit rate in the steady-state. Then from the calibration that follows we can establish that $\frac{\Theta}{(F_E + G_\lambda)^2} << 1$ and $-F_E >> G_\lambda$. Then expanding (42) as a binomial series we have

$$\mu_1 = \frac{F_E + G_\lambda - |F_E + G_\lambda| \left[1 - \frac{\Theta}{(F_E + G_\lambda)^2}\right]^{\frac{1}{2}}}{2} \approx F_E + G_\lambda + \frac{\Theta}{|F_E + G_\lambda|} \approx F_E \quad (51)$$

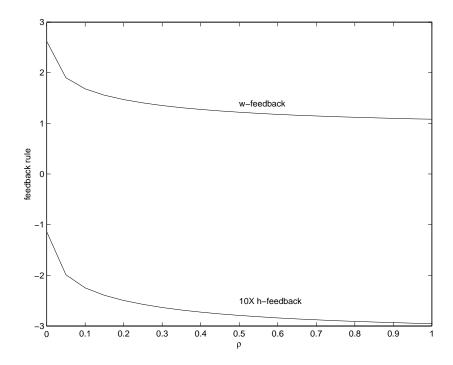


Figure 3: Feedback Rules as ρ changes

From (45) for $r \ll \rho \bar{q}$, \bar{q} , $r \ll E(Eq'' + q')$ and it follows that $\frac{dF_E}{d\rho} \ll 0$. Hence we have established the proposition:

Proposition 4

For $r<<\rho\bar{q},$ an increase in risk aversion increases the degree of stability of the economy.

The intuition for this result can best be seen by examining the decision variables of the firm, trajectories for the wage rate and the hiring rate, $\{w(t), h(t)\}$ in terms of a policy feed-back rule that responds to the current state of employment in the firm and the value of the trained worker. In terms of aggregate employment (assuming all firms are identical), in the vicinity of the steady state $[\bar{w}, \bar{h}]$ the policy rule of the representative firm is is given by

$$\begin{bmatrix} w(t) - \bar{w} \\ h(t) - \bar{h} \end{bmatrix} = \begin{bmatrix} w_E & w_\lambda \\ h_E & h_\lambda \end{bmatrix} \begin{bmatrix} E(t) - \bar{E} \\ \lambda(t) - \bar{\lambda} \end{bmatrix}$$
 (52)

Now consider the firm is adjusting to the steady state along a trajectory where $e(t) > \bar{e}$. For $r << \rho \bar{q}$ we have seen that $\mu_1 \approx F_E$ and hence from (37) $\lambda(t) \approx \bar{\lambda}$.

Then the policy rule takes the approximate form

$$\begin{bmatrix} w - \bar{w} \\ h - \bar{h} \end{bmatrix} = \begin{bmatrix} w_E \\ h_E \end{bmatrix} (e(t) - \bar{e}) = \begin{bmatrix} [(Eq'' + q')T' + Eq'T''h_E] \\ h_E \end{bmatrix} (E(t) - \bar{E})$$
 (53)

substituting for w_E from (30). Now compare a firm with relatively low and high degrees of risk aversion. In the former case h_E is small so the firm responds to being above the equilibrium employment level by raising the wage, but only changes the hiring rate slightly. In the more risk averse case, for r < E(Eq'' + q') (easily satisfied in our calibration), $h_E < 0$ and grows in absolute size as ρ increases, so the wage is raised by less than in the less risk averse case, quits are therefore higher, and now the firm responds by lowering the hiring rate. As a result of these differences employment returns faster to its steady state for the risk averse firm. Clearly adjustment for $e(t) < \bar{e}$ is symmetrical: now the more risk averse firm hires more and engineers less quits than the less risk averse risk neutral firm. In short, firms are self-stabilizing in the face of shocks, and the more risk averse firm will stabilize employment by more than the less risk averse firm. Figure 3 plots the exact feedback coefficients $w_E - M_{22}^{-1} M_{21} w_{\lambda}$ of the wage on employment (the 'wfeedback') and the corresponding 'h-feedback' $h_E - M_{22}^{-1} M_{21} h_{\lambda}$ computed without the approximation used in the analysis, for parameter values discussed in next subsection. The same features revealed in the analysis carry over to the exact solution: the positive w-feedback on employment falls and the negative h-feedback rises in absolute value, increasing the self-stabilization of the firm as risk aversion increases.

4.4 Numerical Illustration and Quantification

To illustrate these results and attempt to quantify their the degree to which risk averse firms self-stabilize employment, we turn to numerical computations. We examine values of the risk aversion parameter $\rho \in [0, 1]$. We assume the following functional forms:

$$q(E) = q_0 E^{\beta}; \beta > 1 \tag{54}$$

$$T(h) = \frac{1}{2}th^2 \tag{55}$$

This leaves parameters Λ , q_0 , β and t. The interest rate r is exogenous and we set its value to a plausible average real interest rate. By choice of units we can normalize $\Lambda=1$ so that a fully employed labour force produces on unit of output. \bar{E} is given by $1-\bar{U}$ where \bar{U} is the natural rate of unemployment for which econometric estimates are available. To calibrate the remaining three parameters we use the steady state relationships given by (17) to (19) and econometric estimates for the unemployment rate, and data for the quit rate q. We need one more item of data to complete the calibration. Data on training costs T(h)E is available, but leads to implausibly low degrees of persistence. We therefore appeal to estimates of the latter to pin down the final parameter.

First we note that from (18), w(E) = Eq'(E) and with our chosen functional forms we have

$$w'(E) = 2\beta^2 q_0^2 t E^{2\beta - 1} > 0 (56)$$

$$w''(E) = 2\beta^2 (2\beta - 1)q_0^2 t E^{2\beta - 2} > 0$$
(57)

Thus the 'wage curve', w(E), has the familiar convex shape.

Now suppose we use data on training costs (as a proportion of total output) $\bar{T}\bar{E} = \overline{TC}$, say, and on quit rate \bar{q} . Then we have that $\frac{1}{2}t\bar{q}^2\bar{E} = \overline{TC}$ and hence t is calibrated as:

$$t = \frac{2\overline{TC}}{\overline{E}\overline{q}^2} \tag{58}$$

Given t and exogenous r, from (19) and $\bar{w} = \beta t \bar{q}^2$ we can solve for β as

$$\beta = \frac{1 - \frac{\overline{TC}}{E} - t\overline{q}r}{t\overline{q}^2} \tag{59}$$

Finally q_0 is now calibrated as $q_0 = \frac{\bar{q}}{\bar{E}^{\beta}}$, though in fact this parameter is not required for the stability analysis. We have completed the calibration of parameters β and t.

 $^{^{10}}$ The annual quit rate in Japan in 1989 was 10.9, for US 19.4, France 10.8 and Spain 9.6 (all in 1991 - see OECD (1994)).

 $^{^{11}}$ The average training cost for 12 European countries as a share of total labour costs is 1.5% and for UK 2.8% (OECD, 1999).

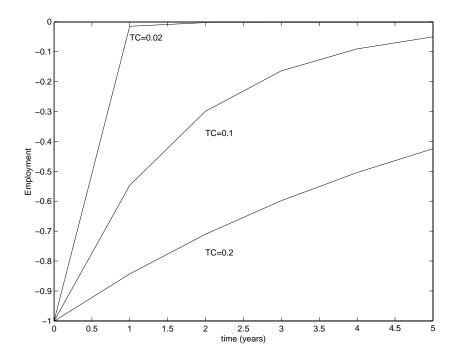


Figure 4: Calibration of Training Costs (TC)

For the computations of μ_1 we then use

$$T' = t\bar{q}; \qquad T'' = t \tag{60}$$

$$T' = t\bar{q}; \qquad T'' = t \qquad (60)$$

$$q' = \frac{\beta\bar{q}}{\bar{E}}; \qquad q'' = \frac{\beta(\beta - 1)\bar{q}}{\bar{E}^2}$$

Then substituting into (45), (46) and (46) we have

$$F_{E} = \frac{\rho \bar{q}(r - \bar{q}\beta^{2})}{(\rho \bar{q}(\beta + 1) + r)} - \beta \bar{q}$$

$$G_{\lambda} = \frac{r[(2 + \beta)\bar{q} + r]}{(\rho \bar{q}(\beta + 1) + r)}$$

$$\Theta = -\frac{\bar{q}\beta r[(2 + \beta)\bar{q} + r]}{(\rho \bar{q}(\beta + 1) + r)} = -\bar{q}\beta G_{\lambda}$$
(62)
$$(63)$$

$$G_{\lambda} = \frac{r[(2+\beta)\bar{q}+r]}{(\rho\bar{q}(\beta+1)+r)} \tag{63}$$

$$\Theta = -\frac{\bar{q}\beta r[(2+\beta)\bar{q}+r]}{(\rho\bar{q}(\beta+1)+r)} = -\bar{q}\beta G_{\lambda}$$
(64)

From these expressions it is clear that if $\rho \bar{q} >> r$, then $\frac{\Theta}{(F_E + G_\lambda)^2} << 1$, justifying the approximations made in the analysis of the previous section.

We can now compute μ_1 using these functional forms and parameters. Figure 4 sets $\rho = 0.5$ and shows the employment response to a unit negative shock (i.e., E(0) – \bar{E} = -1) about the steady state for empirical data \bar{q} = 0.1, \bar{U} = 0.05, $\overline{TC}=0.02$ and r=0.025 and compares the response with those with higher values of \overline{TC} . Clearly using data that suggests $\overline{TC}=0.02$ leads to an implausibly quick responses: the economy returns to equilibrium within a year! However, a 50% recovery within 3 years fits in with the stylized facts¹² and from our graphs this suggests the \overline{TC} should be between 0.1 and 0.2. We choose $\overline{TC}=0.15$. Our data suggest that much of this cost of training does not appear as measured investments and counts as the 'intangible investment', highlighted for instance in Parente and Prescott (2000). Since their estimate of total intangible investment could be as much as 50% of GDP for the US, our figure of 15% for training does not seem excessive.

With these parameter values, increasing risk aversion then increases the absolute size of the negative eigenvalue (figure 5) making the macro-economy more stable, and decreases unemployment persistence following a negative shock to employment (figure 6). The policy rules for the risk-neutral and risk averse firms are contrasted in figure 7. For the risk averse case wages are higher, quits are therefore lower and more hiring takes place confirming the increased self-stabilization of the firm following a negative employment shock. Another way of quantifying the degree of hysteresis is to evaluate the asymptotic variance of employment subject to a white noise shock of variance σ^2 . This is given by $\frac{\sigma^2}{-\mu_1}$. Then the effect of increasing ρ from 0 to 1 is to decrease the variance of employment by a factor $\frac{\mu_1(0)}{\mu_1(1)}$ which turns out to be about $\frac{2}{3}$ in our example.

5 Conclusion

The question we have examined is what explains the delays in the speed of adjustment of unemployment rate towards its steady-state levels. We have shown that risk aversion behaviour in firms tends to speed up the adjustment process towards the steady-sate employment levels as the firms attempt to minimise fluctuations in profits. One clear policy implication stands out: if firms are risk averse there is *less* need

¹²See, for example, Alogoskoufis and Manning (1988) and Henry et al (2000)

for macroeconomic policy stabilization through monetary and fiscal policy. Our calibrated model suggests however that allowing the coefficient of risk aversion increase from $\rho=0$ to $\rho=1$ removes around one-third of employment variation, therefore still leaving a substantial role for government. The topic of how the institutional structure of the economy and the degree of risk aversion in firms are related, and how these may differ across countries to cause different adjustment speeds, awaits further work.

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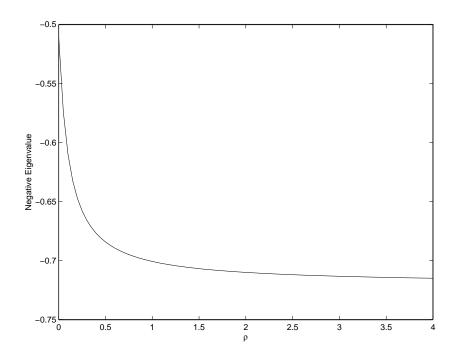


Figure 5: The Negative Eigenvalue as ρ changes

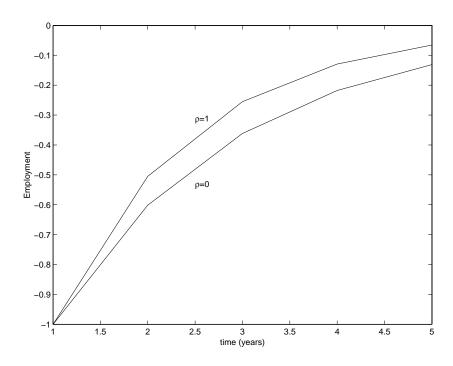


Figure 6: Employment Trajectories as ρ changes

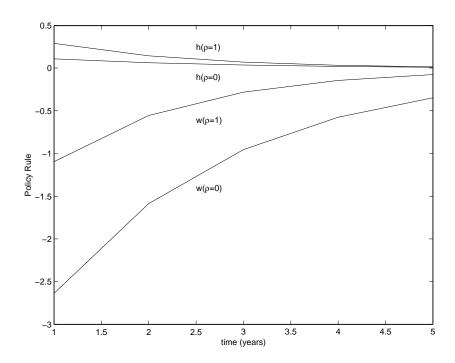


Figure 7: Hiring and Wage Trajectories as ρ changes